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On hypersurfaces of HGF - Manifold

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Abstract

In previous papers and we have studied some properties of hypersurfaces of HGF-manifold. In this paper I have defined hyperbolic almost Kahler manifold and studied its hypersurface. It has been found that the hypersurface of a hyperbolic almost Kahler manifold is locally quasi-Sasakian manifold. Some results regarding the hypersurfaces of a flat HGF-manifold have also been obtained.

Keywords: HGF-manifold, Hypersurfaces, Geometric structures on manifolds

Introduction

Consider a differentiable manifold M_n of class c^∞. Let there be a vector valued linear function F of class c^∞, satisfying the algebraic equation.

(1.1) $F^2 = - a^2 I_n$,

Where ‘a’ is a complex number.

Then F is said to give to M_n a hyperbolic differentiable structure, briefly HGF-structure, defined by algebraic equation (1.1) and the manifold M_n is called HGF manifold [5]. The equation (1.1) gives different algebraic structures for different values of a. If $a \neq 0$, it is a hyperbolic $\pi -$ structure. $a = \pm 1$, it is an almost complex or an almost hyperbolic product structure. $a = \pm i$, it is an almost product or an almost hyperbolic product structure and a = 0, it is an almost tangent structure. in the second case n has to be even and in the second and third case $a^4 = 1$.

If the HGF-structure is endowed with a Hermite metric G.s.t.

(1.2) G (F⋋,Fယ) =, G (⋋,ယ)

Then {F, G} is said to give to M_n hyperbolic Hermite structure c briefly known as HH-structure subordinate to HGF structure.

In a hyperbolic H-structure, if

(1. 3) (E⋋F***) (ယ) = 0***, or (E⋋F***) (F ယ) = 0***,

is satisfied, then M_n is said to be a hyperbolic Kahler manifols, F is the Riemannian connexion.

In a hyperbolic H-structure, if

(1.4) ***(E⋋F***) (ယ) + ***(E ယF) (⋋)*** = 0,

is satisfied, then M_n is said to be hyperbolic nearly Kahler manifold. Let us consider $M_{n \text{ and }} M_m$ as the HH - manifold and its hypersurface respectively.

Let b : $M_m \to M_n$ be the embedding map, s.t. $p \in M_m \to bp \in M_n$.

Let B be the corresponding Jacobian map, s.t. a vector field X in M_m at p· BX in M_n at bp. Let g be the induced Riemannian metric in M_m. Thus we have-

(1.5) ***G(BX, BY) ob = g (X,Y),***

for arbitrary vector fields X,Y in M_M**.**

(1.6) *a* *G (N,N) ob* = 1**,**

(1.6) b G (N,BX) ob = 0**,**

for a unit normal to M_m.

If we put

(1.7) a FBX = B (fX) + u (X) N,

(1.7) b FN = - BU.

Then it can be easily seen that

(1.8) a $\bar{\bar{x}} = - a^2 X + u(X) U$,

(1.8) b u (fX) = O,

(1.8) c $u(U) = - a^2$,

(1.8) d fU = 0 and

(1.9) $g(\bar{X}, \bar{Y}) = a^2 g(X, Y) - u(X) u(Y)$,

Where,

X = fX and $U(X) = g\,\overline{(X}, \underline{U})$

i.e. the induced structure is a general contact metric structure.

Let E and D be the Riemannian connexions in M_n and M_m respectively. Gauss and Weingarten equations are :

(1.10) a $E_{BX}BY = BD_XY$ + ‘H (X, Y) N,

(1.10) b $E_{BX}N$ = - BHX, respectively [1].

Where,

‘H (X, Y) = g (HX, Y).

Let R and k denote the curvature tensors with respect to the connexions E and D respectively. The Generalised Gauss and Mainardi – Codazzi are given by [5] :

(1.11) a ‘R (BX, BY, BZ, BW) ob = ‘K (X, Y, Z, W) + a^2 ‘H (X, Z) ‘H (Y, W) -

a^2 ‘H (Y, Z) ‘H (X, W).

(1.11) b ‘R (BX, BY, BZ, N) ob = a^2{ (D_X ‘H) (Y, Z) - (D_Y ‘H) (X, Z) }.

Where, ‘R (BX, BY, BZ, BW) = G(R(BX,BY,BZ), BW).

On the hypersurface of a hyperbolic Kahler manifold subordinate to HGF- manifold the following results hold [2] :

(1.12) a $(D_Xf)Y$ = u (Y) HX – ‘H(X, Y) U

(1.12) b $(D_Xu)(Y)$ = - ‘H (X, Y).

Agreement (1.1) in the above and in sequal ⋋, ယ,v,.. will be taken as an arbitarary vector feilds in the enveloping manifold and X,Y,Z,............................... as arbitrary vector fields in the hypersurface.

Hyperbolic Almost Kahler Manifold :

Definition ***(2.1).*** Hyperbolic Hermite manifold satisfying

(2.1) *a* $(E_\lambda {}'F)(\mu, \nu) + (E_\mu {}'F)(\nu, \nu) + (E_\nu {}'F)(\lambda, \mu) = 0$,

Where,

$'F(\lambda, \mu) = G(F\lambda, \mu)$.

Will be called hyperbolic almost Kahler manifold, subordinate to HGF – manifold

From the equation (***1.7)*** a, we have

(2.1) b $G(FBX, BY) = G(BfX, BY) + u(X)\, G(N, BY)$

Differentiating equation (2.1) b, covariantly w.r. to BZ, then using the equation s (1.5) (1.6) (1.7) and (1.10) a, we have

(2. 2) $(E_{BZ}{}'F)(BX, BY)$ ob $= (D_Z{}'f)(X, Y) + {}'H(X, Z)\, u(Y) - {}'H(Y, Z) u(X)$.

Writing two other equations by cyclic permutations of X, Y, Z we have

(2.3) $(E_{BY}{}'F)(BZ, BX)$ ob $= (D_y{}'f)(Z, X) + {}'H(Z, Y)\, u(X) - {}'H(X, Y) u(Z)$

and

(2.4) $(E_{BX}{}'F)(BY, BZ)$ ob $= (D_X{}'f)(Y, Z) + {}'H(Y, X)\, u(Z) - {}'H(X, Z) u(Y)$

Thus, we have the following theorem: Theorem (2. 1). If the enveloping manifold is a hyperbolic almost Kahler manifold, its hypersurface is given by

(2.5) $(D_X{}'f)(Y, Z) + (D_Y{}'f)(Z, X) + (D_Z{}'f)(X, Y) = 0$

Proof. Adding the equations (2.2) (2.3) (2.4), we have

(2.6) $\{ E_{BZ}{}'F)(BX,BY) + (E_{BY}{}'F)(BZ, BX) + (E_{BX}{}'F)(BY, BZ) \}$ ob =

$(D_X{}'f)(Y, Z) + (D_Y{}'f)(Z, X) + (D_Z{}'f)(X, Y)$

Using the equation (2.1)a in the equation (2.6), we get the equation (2.5).

Corollarly (2.1). Hypersurface of Hyperbolic almost Kahler manifold is locally quasi-sasakian manifold.

Proof. Equation (2.5) Proves the statement.

Theorem (2.2). For the hypersurface of a hyperbolic almost Kahler manifold, we have

(2.7) $(D_X{}'f)(Y, Z) + (D_Y{}'f)(Z, X) + (D_Z{}'f)(X, Y) + {}'f((D_Zf)X - (D_Xf)Z, Y))$
$+ {}'f((D_Yf)Z - (D_Zf)Y, X)) + {}'f((D_Xf)Y - (D_Yf)X, Z)) = 0$

Proof. We have

(2.8) a $'f(X,Y) = g(fX, Y) = {}'f(Y, X)$,

Differentiating (2. 8) b covariantly w.r. to Z and Using the equation (2. 8) again, we get

(2.9) a $(D_Z{}'f)(X, Y) + {}'f((D_Zf)X, Y)) + {}'f((X, (D_Zf)Y)) = a^2(D_Z{}'f)(X, Y)$.

Similarly, writing two other equations, we have

(2. 9) b $(D_Y{}'f)(Z, X) + {}'f((D_Yf)Z, X)) + {}'f((Z, (D_Yf)X)) = a^2(D_Y{}'f)(Z, X)$,

(2. 9) c $(D_X{}'f)(Y, Z) + {}'f((D_Xf)Y, Z)) + {}'f((Y, (D_Xf)Z)) = a^2(D_X{}'f)(Y, Z)$,

Adding the Equations (2. 9) a, b and c, then using the equations (2. 8) a and (2.5), we get the required result.

Hypersurfaces of Flat HGF-Manifold:

Theorem (3.1) The umbilical Hypersurface of a Hyperbolic General Differentiabe (HGF) manifold is i=of constant Riemannian curvature, if the enveloping manifold is flat.

Proof. Let the hypersurface be umbilic, i.e.

$'H(X, Y) = g(X, Y)$, [1 }

Then (1.11) a, gives

(3.1) $'R(BX, BY, BZ, BW)$ ob $= {}'K(X, Y, Z, W) + a^2g(x, z)g(Y, W) - a^2g(Y,Z)g(X,W)$

If the enveloping manifold is flat, then (3.1) reduces to

(3.2) $'K(X, Y, Z, W) = a^2\{g(Y, Z)g(X,W) - g(X, Z)\, g(Y, W)\}$

This shows that the hypersurface is of constant Riemannian curvature.

Conversely, if (3.2) holds, then using (3.2) in (3.1) we have $'R(BX, BY, BZ, BW) = 0$, that is the manifold is flat.

Theorem (3.2). The scalar curvature of the umbilical hypersurface M_m of a flat HGF – manifold M_n is given by

(3.3) $r = m(m-1)a^2$

Proof. The umbilical hypersurface is of constant Riemannian curvature (by theorem (3.1)). We have

$K(X, Y, Z) = a^2\{ g(Y, Z)X - g(X, Z)Y \}$

From this we at once get the equation (3.3).

Theorem (3.3) The quasi – umbilical hypersurface of a flat HGF – manifold can never be of constant Riemannian curvature.

Proof. Let the hypersurface of a flat HGF – manifold be quasi –umbilical, then w can always write

(3.4) $'H(X, Y) = g(X, Y) + u(X)u(Y)$, [1]

Using (3.4) in (1.11) a, we have

(3.5) $'R(BX,BY,BZ,BW)$ ob $= {}'K(X,Y,Z,W) + a^2g(X,Z)g(Y,W) - a^2g(Y,Z)g(X,W) + a^2g(Y,W)u(X)u(Z) + a^2g(X,Z)u(Y)u(W) - a^2g(Y,Z)u(X)u(W) - a^2g(X,W)u(Y)u(Z)$

Now,

$'K(X, Y, Z, W) = a^2\{g(Y,Z)g(X,W) - g(X, Z)\, g(Y, W)\}$,

if

$a^2\{ g(Y,W)\, u(X)\, u(Z) + g(X, Z)u(Y)u(W) - g(Y,Z)u(X)u(W) - g(X,W)u(Y)u(Z)\} = 0$

Let $a^2 \neq 0$, then

$g(Y,W)u(Z)u(Z) + g(X,Z)u(Y)u(W) - g(Y,Z)u(X)u(W) - g(X,W)u(Y)u(Z) = 0$

or

$g(Y,W)u(X)U + u(Y)u(W)X - u(X)\, u(W)\, Y - g(X,W)u(Y)U = 0$,

or

$a^2g(Y,W) + mu(Y)u(W) - u(Y)u(W) - u(Y)u(W) = 0$

or

$a^2g(Y,W) + (m-2)u(Y)u(W) = 0$,

or

$a^2Y + (m-2)u(Y)U = 0$,

or

$ma^2 + (m-2)a^2 = 0$, or $2a^2(m-1) = 0 \Rightarrow a^2 = 0$,

But $a^2 \neq 0$, thus

The quasi-umbilical hypersurface cannot be of Constt. Riemannian curvature.

Theorem (3.4) If the hypersurface of a flat HGF-manifold is of minimal variety,
then divH=0.
But the converse is not true in general.
Proof. Let the hypersurface be of minimal variety then
tr.H = 0, [1]
Since the enveloping manifold is flat, equation (1.11) b implies that
$(D_X H)Y - (D_Y H)X = 0$
Contracting this equation, we get
(divH) Y = Y tr.H= 0 (Since Tr.H = 0)
Conversely, if div H = 0, then from the last equation, we get Ytr.H = 0 i.e. tr.H = constt.

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